

# TRAJECTORY DESIGN FOR THE MICROWAVE ANISOTROPY PROBE (MAP)

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**ABSTRACT** – The Microwave Anisotropy Probe (MAP) is a Medium Class Explorers (MIDEX) mission produced in partnership between Goddard Space Flight Center (GSFC) and Princeton University. The goal of the MAP mission is to produce an accurate full-sky map of the cosmic microwave background temperature fluctuations (anisotropy). The mission orbit is a Lissajous orbit about the  $L_2$  Sun-Earth Lagrange point. The trajectory design for MAP is complex, having many requirements that must be met including shadow avoidance, sun angle constraints, Lissajous size and shape characteristics, and limited Delta-V budget. In order to find a trajectory that met the design requirements for the entire 4-year mission lifetime goal, GSFC Flight Dynamics engineers performed many analyses, the results of which are presented herein. The paper discusses the preliminary trade-offs to establish a baseline trajectory, analysis to establish the nominal daily trajectory, and the launch window determination to widen the opportunity from instantaneous to several minutes for each launch date.

**KEYWORDS:** MAP, trajectory, Lissajous, launch window

## INTRODUCTION

The Microwave Anisotropy Probe (MAP) is a Medium Class Explorers (MIDEX) mission produced in partnership between Goddard Space Flight Center (GSFC) and Princeton University. The goal of the MAP mission is to produce an accurate full-sky map of the cosmic microwave background temperature fluctuations (anisotropy). These data will shed light on several key questions related to the Big Bang theory and expand on the information provided by the National Aeronautics and Space Administration (NASA) Cosmic Background Explorer (COBE) mission.

MAP was launched on June 30, 2001 at 19:36:36.183 Z aboard a Delta II 7425 expendable launch vehicle (ELV) from the Eastern Range. The injection orbit was a 185 km highly elliptical parking orbit with a 28.7° inclination. The mission orbit is a Lissajous orbit about the  $L_2$  Sun-Earth Lagrange point, about 1.5 million km from Earth in the anti-Sun direction. This orbit was selected to minimize environmental disturbances and maximize observing efficiency. Mission duration is approximately 27 months, with 3

months of transfer time to the  $L_2$  orbit. The transfer was accomplished using a series of phasing loops and a lunar gravity assist. Once in orbit about  $L_2$ , the spacecraft will maintain a Lissajous orbit such that the MAP-Earth vector remains between  $0.5^\circ$  and  $10.5^\circ$  off the Sun-Earth vector to satisfy communications requirements while avoiding eclipses. MAP is the first spacecraft to use an orbit around the  $L_2$  point as its permanent observing station. It will take about 18 months to build up a full-sky picture and for scientists to perform the analysis.

This paper discusses the design of the MAP Trajectory to meet mission requirements, including discussion of preliminary trade-offs to establish a baseline trajectory plan, analysis to establish the nominal daily trajectory, and the launch window determination to widen the opportunity from instantaneous to several minutes for each launch date. Contingency analysis and statistical verification of methods are considered in other publications.

## INITIAL TRAJECTORY DESIGN

The requirements for the MAP trajectory as defined by the flight project include:

- Perigee heights above 500 km
- Delta-V budget for the phasing loops not to exceed 70 m/s
- Pf shall be less than 30 m/sec
- MAP-Earth vector between  $0.5^\circ$  and  $10.5^\circ$  off the Sun-Earth vector
- Two-year lifetime in the Lissajous
- No Earth shadows in the Lissajous

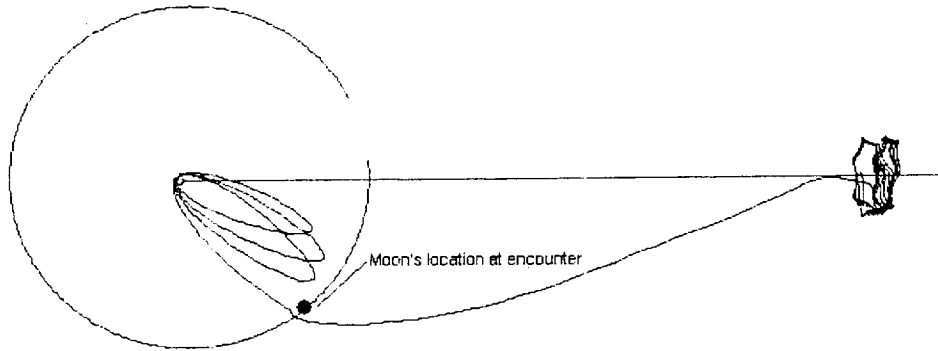
Other mission goals that the nominal mission trajectories had to be designed to meet included:

- Four year lifetime in the Lissajous
- Minimize moon shadows in the cruise and Lissajous phases

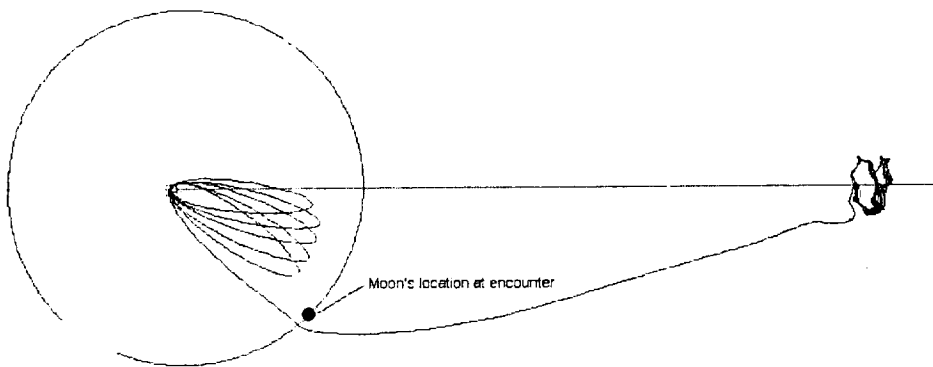
Meeting both the requirements and the goals proved quite challenging. The initial conditions provided by the Delta in the injection orbit were a C3 of  $-2.6 \text{ km}^2/\text{sec}^2$ , a mass to orbit of 831 kg, and a 185-km perigee. In order to attain a Lissajous orbit about  $L_2$  from this injection orbit, two types of transfer trajectories were considered: direct from injection to a lunar swingby; and a zero-cost insertion using phasing loops prior to a lunar swingby. The direct ascent approach was not a viable option because it required 150 to 200 meters per second of delta-V, which not only exceeded the 70 meter per second fuel allotment for the phasing loops but required using the majority of the fuel in the tank. Therefore, the phasing loop option was chosen in order to use the moon to achieve a Lissajous orbit while staying within the allotted fuel budget. The idea of the phasing loops is to adjust the timing of the lunar gravity assist by performing delta-V maneuvers at appropriate perigee passes in order to raise the final apogee to lunar orbit distance. Phasing loops are considered operationally flexible and low risk in that there are multiple opportunities to perform maneuvers and hone the lunar swingby. The next task was to determine the number of phasing loops to use in the trajectory design.

Initially, a traditional design using either 2 or 4 loops (the Wind and Geotail missions each used 4 loops) was examined. However, the 2-loop scenario was considered risky in that there were few maneuver opportunities, all of which would be critical to the success of the mission. Four phasing loops were not an option because lunar perturbations had undesirable effects on the trajectory design. Prior analysis performed by CSC had indicated that using three and five phasing loops as a baseline would be the best approach [1]. There is a natural transition from a 3-loop to a 5-loop trajectory because of the location of the moon in its orbit relative to the timing of the apogees in the loops. Examples of 3- and 5-loop trajectories are shown below in Figures 1 and 2, respectively. In both figures, the large circle indicates

the moon's orbit. The next step was to design the phasing loops such that a Lissajous orbit could be obtained that met all the mission requirements.



**Fig. 1. Example of a 3-loop trajectory with lunar swingby to a Lissajous orbit at L2.**



**Fig. 2. Example of a 5-loop trajectory with lunar swingby to a Lissajous orbit at L2.**

The shape of a Lissajous orbit is characterized by its amplitude and phase. The Lissajous orbit for MAP was designed to meet the Sun-Earth-Vehicle (SEV) angle requirement for the mission as well as to minimize lunar shadows. The SEV angle was to remain between 0.5- and 10.5-degrees in order to maintain adequate communications link margins. In addition, there was a desire to avoid lunar shadows for the duration of the two-year mission due to thermal considerations. The SEV behavior is dependent on both the phase and the amplitude of the orbit. Additionally, the phase also affects the occurrences of lunar shadows. This necessitated controlling both the amplitude and phase of the Lissajous orbit.

The amplitude is controlled by several variables. P. Sharer, et al. demonstrated in their analysis [2] that the lunar encounter epoch could be varied to achieve a Lissajous orbit with a given amplitude, but the amplitude is also a function of the spacecraft's energy at the encounter. The encounter epoch defines the encounter angle, which is the angle from the Earth-Sun line rotated clockwise to the Earth-Moon line at the time of the encounter, often referred to as the Sun-Earth-Moon (SEM) angle. The encounter angle, in turn, helps determine families of launch and coast solutions since they are strongly correlated to the encounter time. The energy is driven, in part, by the delta-V maneuvers that are performed at the perigees of the phasing loops. The phase of the Lissajous orbit is strongly affected by the B-plane parameters.

namely B•R and B•T. Hence, the main trajectory design drivers were the encounter angle, the energy, and the B-plane targeting parameters.

The desired range of encounter angles was determined from an analytic model that used various values of the amplitude and phase of a Lissajous orbit chosen based on the SEV constraint. MAP's Lissajous orbit has a relatively small amplitude due to the SEV requirement. For this reason, the lunar encounter was chosen to occur on the trailing edge of the moon as opposed to the leading edge. If the encounter were to take place on the leading edge, the perturbations would be much greater and it would have been difficult to obtain a small amplitude Lissajous while meeting the fuel constraints. The desired encounter angle range was determined to be between 125 and 135 degrees, which fixes the desired Sun-Earth-Moon geometry. Conversely, the B-plane targeting and delta-V perigee maneuvers, which will be revisited in the next section, vary for each trajectory. Once the desired Sun-Earth-Moon geometry had been determined, the development of daily launch trajectories could proceed. The next step was to define the days each month for which a viable trajectory existed that met all of these requirements.

## NOMINAL DAILY TRAJECTORIES

The ultimate goal of computing the daily trajectories was to provide input to Boeing's Detailed Trajectory Objective for the Delta ELV trajectory planning. Therefore, the first step in establishing a nominal daily trajectory was to find a trajectory that worked for a given date. For each day, an encounter time could be determined from the calculated encounter angle. The characteristics of the phasing loops (period, apogee heights, etc.) could then be backed out fairly easily based on whether a 3-loop or 5-loop trajectory was being designed. Due to the orientation of the moon in its orbit with respect to the occurrences of the apogees, 3-loop trajectories were used early in the month and 5-loop trajectories were used late in the month. Since each loop takes about 7-days to complete, the 3-loop trajectories were approximately 24-27 days long from launch until lunar encounter while the 5-loop scenarios were 38-40 days in duration. The goal was to establish a family of launch-coast solutions that would orient the line of apsides such that the right ascension gave the correct SEM angle at the encounter and the declination would lie in or near the lunar orbit plane.

It is appropriate to talk more about the coast time at this juncture. In addition to meeting the spacecraft requirements, Boeing prefers that the coast time for the trajectories in each launch block be the same wherever possible, since Boeing must run a different powered flight design for each coast time. Although many launch-coast combinations exist, only two possible solutions allow the mission requirements to be met. This results from the fact that the launch site rotates through the desired trajectory plane twice daily. One solution is referred to as the "short" coast and the other the "long" coast [1]. The coast time used depends on the time of the year; short coasts are used from January through June, while long coasts are used for July through December. If the wrong coast time is used for a given time of year, Earth shadows will be present in the trajectory.

Once the daily coast time and encounter time were fixed, analytic models were run to seed the targeting software with the appropriate delta-V distributions across perigee maneuvers to achieve the proper timing and energy for the lunar encounter. Finally, B-plane targeting was used to vary the phase of the Lissajous orbit in an attempt to minimize lunar shadows over the two-year mission duration. The B-plane is defined as being perpendicular to the incoming trajectory plane and allows for gravity assist targeting. By targeting the vector along the intersection of the B-plane and trajectory plane (B•T) and the vector perpendicular to B•T (B•R), the resulting outgoing trajectory can be altered which will to change the phase of the Lissajous orbit [3]. Some preliminary analysis was done to determine how changing the B-plane parameters affect the resulting phase of the Lissajous. Although, clearly, these parameters could be used to manipulate shadows in the Lissajous, no clear pattern emerged that could be used to predict B•T and B•R values that would minimize lunar shadows. This topic is the subject of ongoing analysis by the MAP mission trajectory analysts and will be presented at a later date. Once the trajectory was balanced, it was examined to assure that all mission requirements were met. This valid trajectory design would become the "nominal" trajectory for that particular day.

Once the trajectory was balanced, it was examined to assure that all mission requirements were met. This valid trajectory design would become the "nominal" trajectory for that particular day. The next step was to take the nominal trajectory and apply the contracted launch vehicle dispersion of  $\pm 11.6$  m/s ( $\pm 3$ -sigma, determined by Boeing) to see whether redesigning the trajectory to accommodate the dispersions could be accomplished within the budget while meeting all other design requirements. A launch date which had workable nominal and  $\pm 3$ -sigma trajectories was considered a viable launch day. Dates for which there was a nominal trajectory but for which the 11.6 m/s dispersion could not be accommodated were not considered viable. Next, the day before and after the current day were examined in the same way, keeping the encounter time the same. This process continued until a day violated the delta-v constraint. In general, 5 or 6 launch days existed each month for 3-loop cases and 4 or 5 days for 5-loop cases. Whenever possible, the coast time was held constant from day to day. However, if there were problems with many shadows or a closing Lissajous, coast time was opened as a variable to achieve a better Lissajous. Baseline trajectory insertion point orbit elements for each viable launch date were transmitted to Boeing as DTO input.

## LAUNCH WINDOW DETERMINATION

For trajectories that did not require the maximum phasing loop fuel allotment of 70 m/s (either for the baseline or the  $\pm 3$  sigma cases), there was freedom to widen the launch window for that day. Launch windows were computed for four launch blocks: the July 3-loop and 5-loop launch blocks [34] and the August 3-loop and 5-loop launch blocks. The "baseline" trajectory that had been designed for each day and submitted to Delta as DTO input was simply a trajectory that met all of the mission requirements. There was no optimization performed to ensure a minimum fuel budget or the fewest number of shadows. This was because there was no tool available to perform such an optimization, and time to perform analysis or build a new tool was not available prior to launch. Therefore, the baseline trajectory was not necessarily the opening time of the window. In order to determine the window opening and closing times, the launch time of the baseline trajectory was varied in 4- or 5-minute increments forward and backward of the baseline time.

At each time step, the trajectory was retargeted to achieve the lunar encounter conditions attained for the baseline trajectory by varying the phasing loop delta-V's. The targets were some combination of the B-plane values (B•R and B•T) and the C3 at the mid-course correction maneuver (MCCM). Once the lunar encounter was achieved, the resulting trajectory had to be fine-tuned using stationkeeping maneuvers to obtain a balanced Lissajous for the mission lifetime.

Each trajectory was then evaluated against the list of requirements in order to determine the acceptability of that case. If all the criteria were met, this time was considered a valid launch time. Once all the cases for a particular day were analyzed, (nominal and  $\pm 3$ -sigma for each time step), the data was examined to determine the minimum launch block that met all the criteria. This was considered the accepted launch window. A summary of the launch window times for the July launch block that met all mission requirements is provided in Table 1.

Table 1: Launch Window Summary for July Launch Block (2-Year Mission Lifetime) [45]

Date	LW start	LW end	LW in minutes
June 30 <sup>th</sup>	19:46:46	19:56:46	10
July 1 <sup>st</sup>	19:40:11	19:50:11	10
July 2 <sup>nd</sup>	19:34:53	19:49:53	15
July 3 <sup>rd</sup>	19:30:23	19:35:23	5
July 4 <sup>th</sup>	19:20:17	19:35:17	15
July 5 <sup>th</sup>	19:24:36	19:44:36	20
July 16 <sup>th</sup>	20:23:59	20:33:59	10
July 17 <sup>th</sup>	20:18:59	20:43:59	25

July 18 <sup>th</sup>	20:13:57	20:38:57	25
July 19 <sup>th</sup>	20:14:36	20:34:36	20

The launch window for each day is represented graphically in the figures below. All cases achieved a minimum 5-minute launch window, but some cases achieved up to 25 minutes. In cases where  $P_f$  exceeded the 30 m/s limit, a re-design was done to hold  $P_f$  at 30 m/s and use other maneuvers to compensate. In these cases, the total delta-v costs were higher than if the trajectory was designed with no constraint on  $P_f$ . There are a few cases, particularly on July 5<sup>th</sup> and July 19<sup>th</sup>, where the total delta-V costs do exceed 70 m/s by a few meters per second, but were kept anyway in order to increase the launch window for the dates in question. It was decided that increasing the odds of launching on a particular day was more important than strictly adhering to the somewhat arbitrary 70 m/s limit given that there was some contingency fuel allotted in the budget. Also, in some cases, lunar shadows appeared in the cruise phase, although they were not present in the baseline trajectory. There was no attempt to adjust the trajectory in order to eliminate these shadows since the probability that the actual trajectory would have the same shadows was very small (although elimination in most instances would not be difficult or expensive).

The solid lines on the plot indicate times for which all launch window criteria were met. Dashed lines represent launch window cases that did not meet at least one of the criteria. The gray box is the actual launch window, which is the span over which all three cases (nominal and  $\pm 3$ -sigma) met the criteria. Launch windows were restricted in every case by one of two criteria: either the SEV angle was too large (greater than 10.5 degrees) or there were Earth shadows in the Lissajous (a result of the SEV angle being too small).

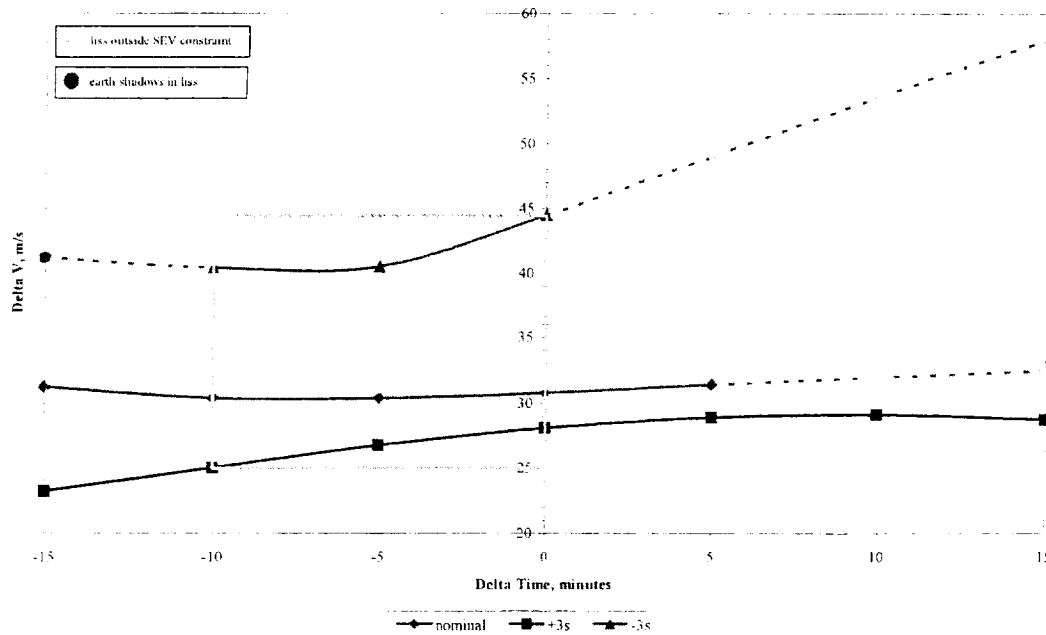


Fig. 3. June 30 Launch Window Results [34]

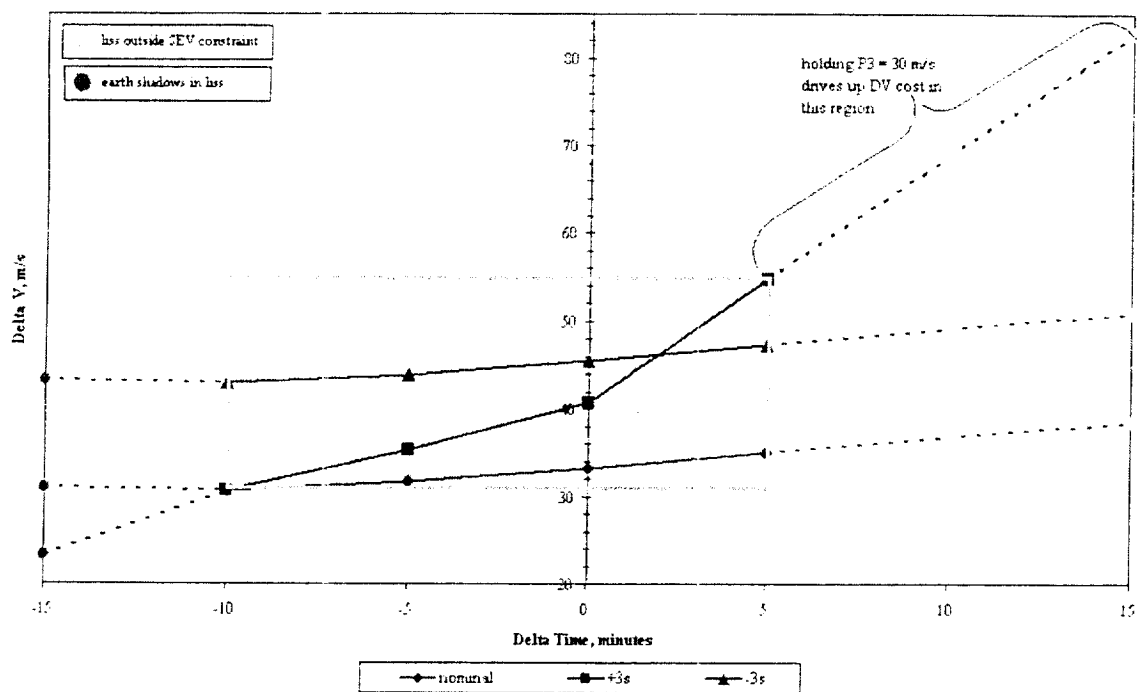


Fig. 4. July 2 Launch Window Results [34]

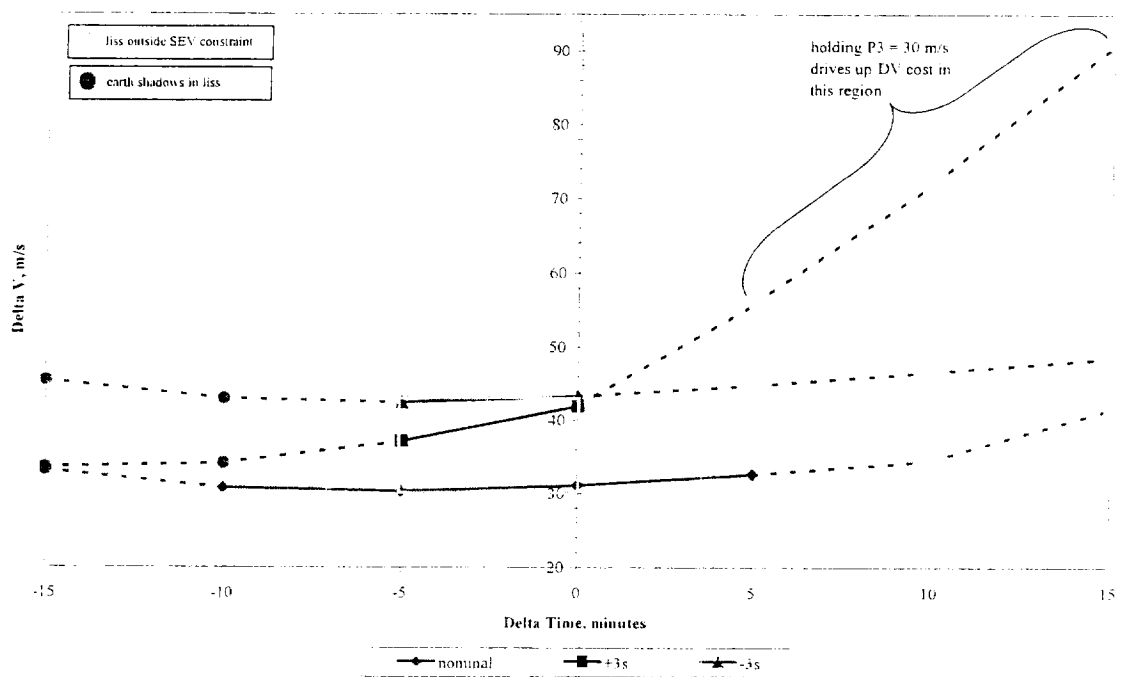


Fig. 5. July 3 Launch Window Results [34]

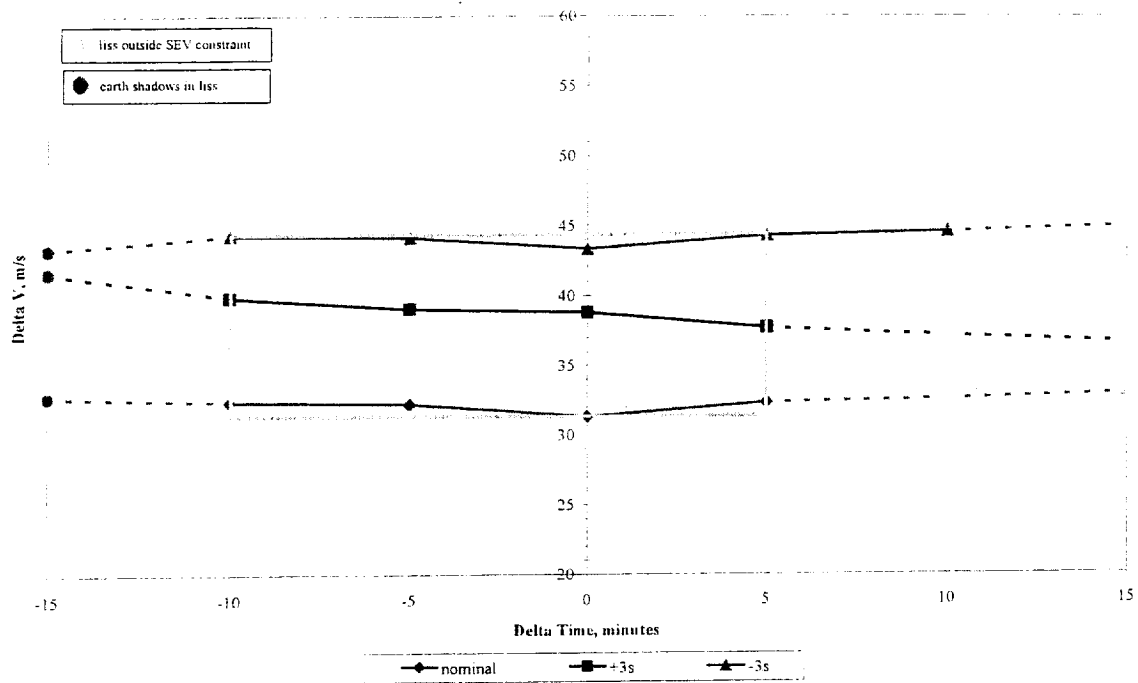


Fig. 6. July 4 Launch Window Results [34]

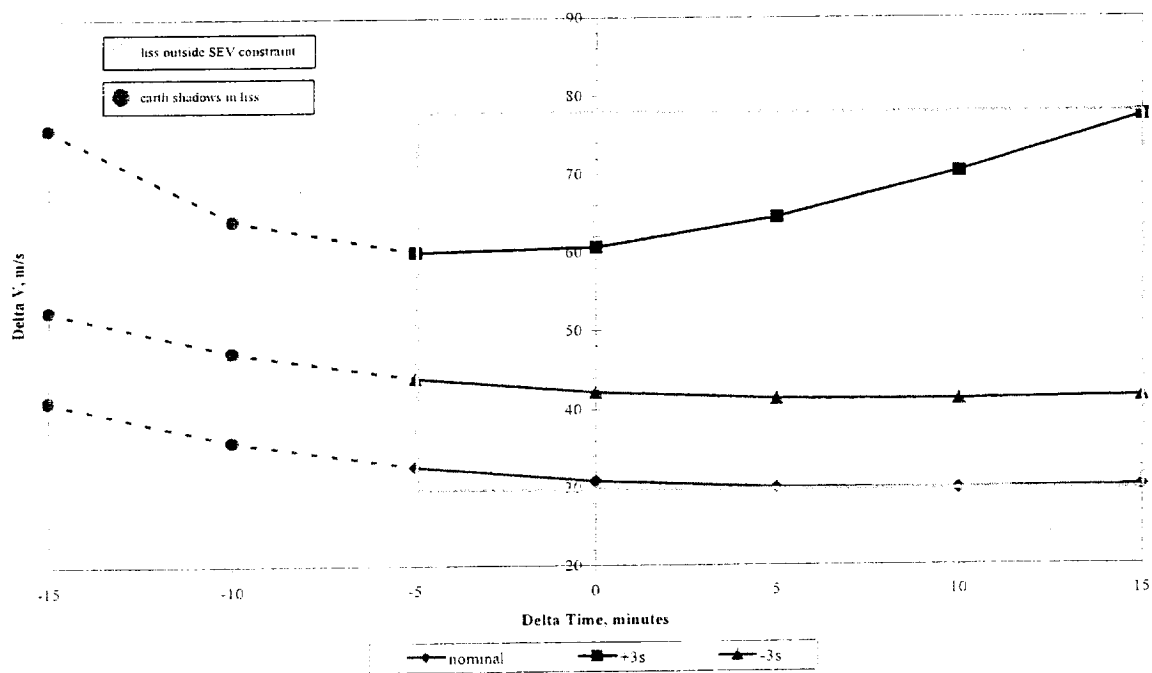


Fig. 7. July 5 Launch Window Results [34]



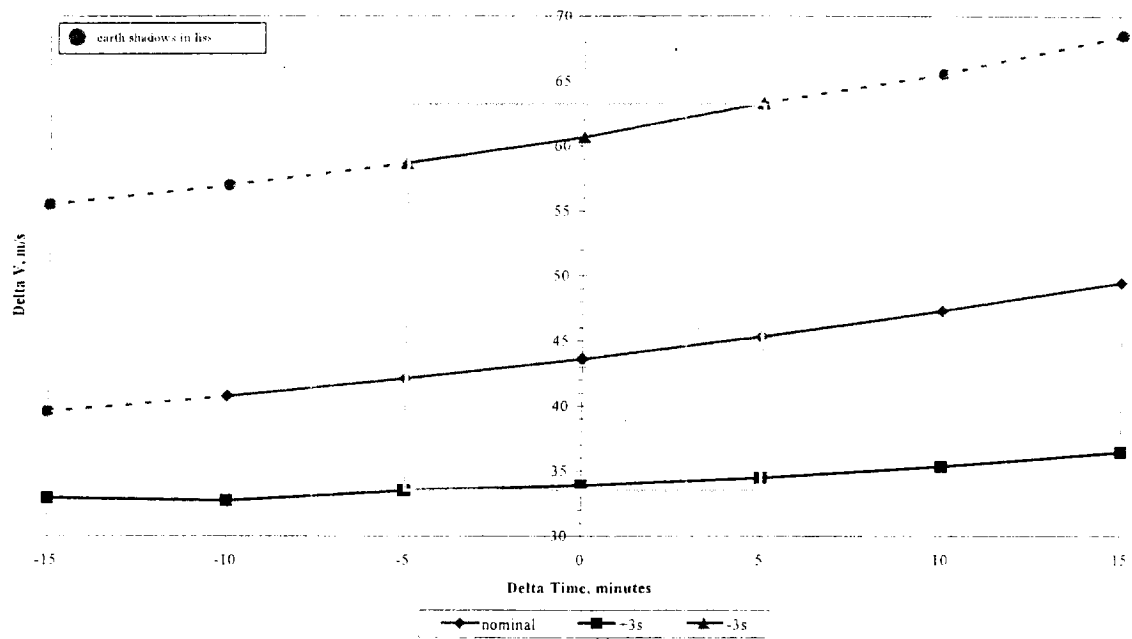


Fig. 8. July 16 Launch Window Results [34]

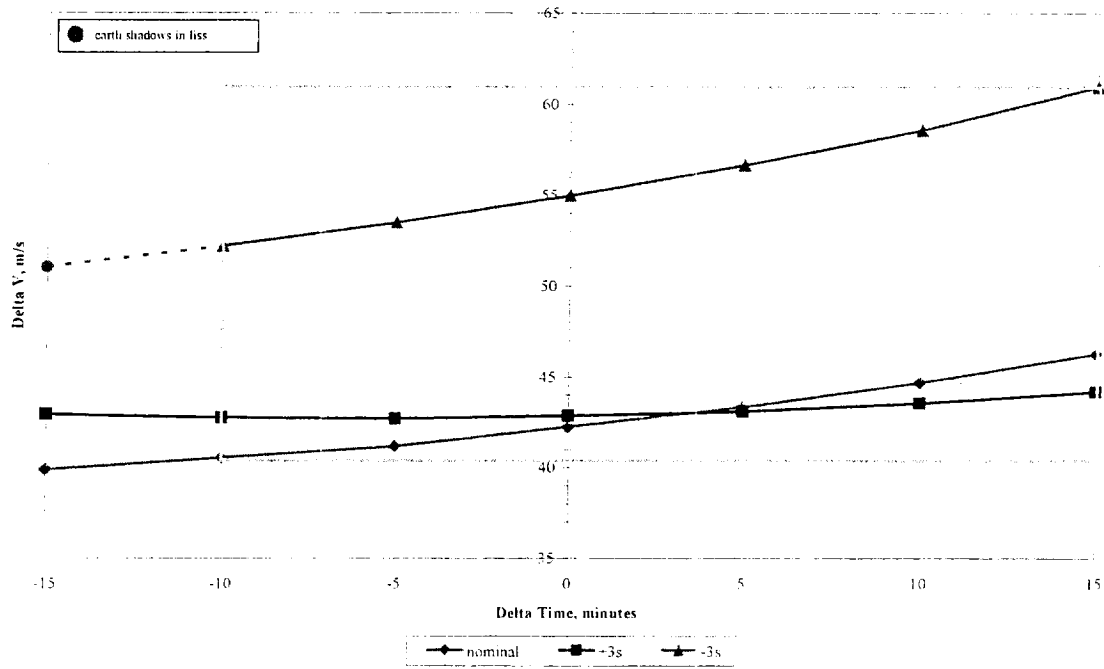


Fig. 9. July 17 Launch Window Results [34]

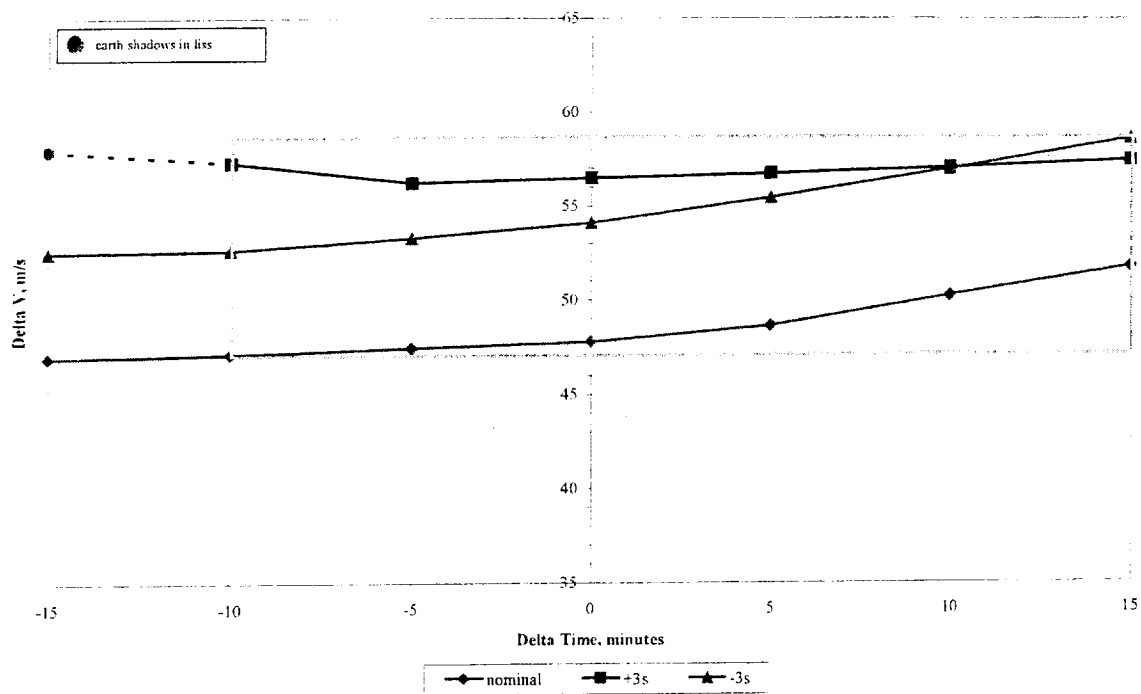


Fig. 10. July 18 Launch Window Results [34]

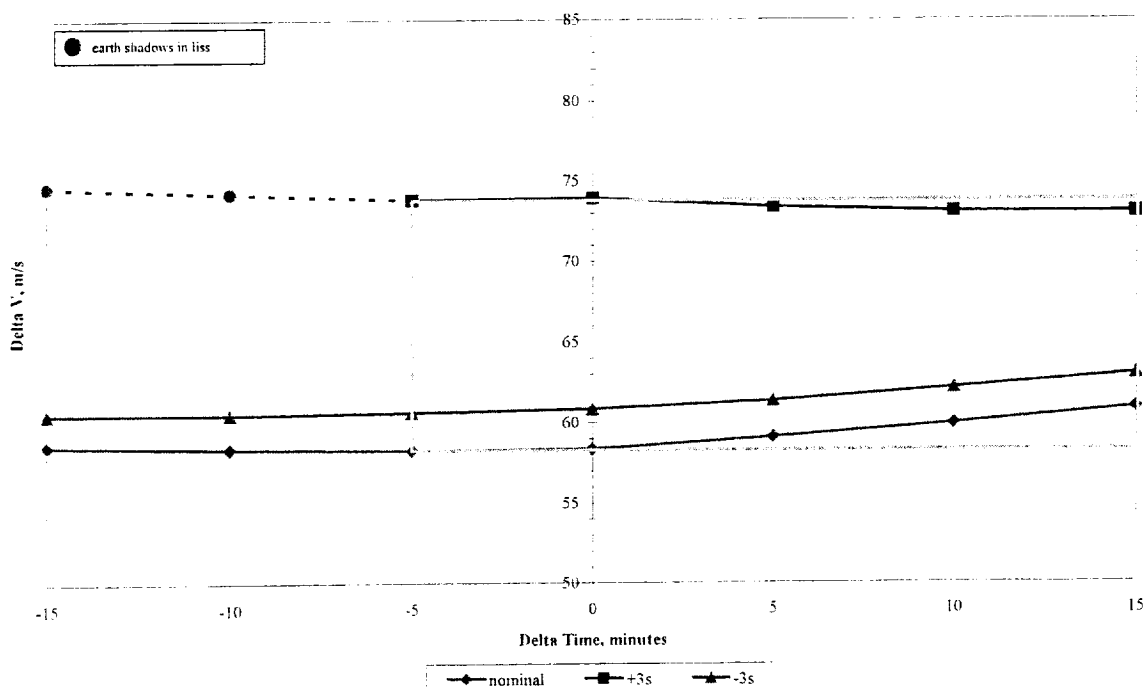


Fig. 11. July 19 Launch Window Results [34]

The July launch window analysis was initially performed assuming a two-year mission lifetime, which is the stated mission requirement. However, the analysis was later expanded to ensure that requirements were met over a four-year lifetime to accommodate a desire to plan for an extended mission.

In the new analysis, the need to avoid Earth shadows after the first two years necessarily required that the 4-year windows be the same or smaller than the corresponding 2-year windows. In the original analysis, it was demonstrated that the Lissajous parameters (i.e. shape and size) could essentially be controlled to avoid Earth shadows by adjusting the baseline orbit to select the appropriate values for the encounter epoch and B-plane targets. This 're-targeting' enabled attainment of a Lissajous size and shape more conducive to extending the mission, while meeting all the constraints. These Lissajous orbits are commonly referred to as "opening" and they allow longer periods in the Lissajous without Earth shadow interference. However, the freedom available in the original trajectory analysis could only be obtained by varying launch time, coast time, and the perigee delta-Vs together. In the case of this launch window analysis, the coast time is fixed by the DTO inputs; thus, this degree-of-freedom is lost, resulting in a shortened launch window. If the launch window for a given day was less than 5 minutes, the trajectory was retargeted using a different value of B•R to change the phase and try to widen the window without encountering shadows.— Dave, how does this relate to the sentence about July 19 below that says "Additional analysis was not performed..."—you did the analysis, right?

It should be noted that a 4-year mission was achieved for the first four days in the block (although the July 2<sup>nd</sup> date does have a smaller window, the shadow does not occur until the very end of the 4-year mission). The last day of the block, July 19<sup>th</sup>, was most affected by attempting to extend the mission to 4 years. For the +3-sigma case the launch window becomes essentially instantaneous (20 seconds). Earth shadows encountered ranged from 1-1/2 to 3-1/2 days in length with depth of shadow ranging from 5% to 15%. ~~Additional analysis was not performed to determine if these Earth shadows could be designed out, since such an analysis would have been too involved and would have impacted launch preparations for the trajectory team.~~ For this particular case, the trajectory was re-designed using a different value of B•R in order to obtain a launch window that was not instantaneous. The B•R value was adjusted and the trajectory retargeted until a sufficient launch window could be defined. A 2-minute launch window was successfully constructed using this method. July 19<sup>th</sup> was the only day where for which this procedure was deemed necessary. – The flight project made a decision to go with the larger launch window to increase chances of getting off the ground in July despite the risk of encountering Earth shadows during the last two years of the mission, since the required mission lifetime is two years.

**Table 2. Launch Window Summary for July Launch Block (4-Year Mission Lifetime) [45]**

Date	LW start	LW end	LW minutes	Earth Shadow Occurrence
June 30 <sup>th</sup>	19:46:46	19:56:46	10	none
July 1 <sup>st</sup>	19:40:11	19:50:11	10	none
July 2 <sup>nd</sup>	19:34:53	19:45:53	10	After 3-3/4 years for –3-sigma case
July 3 <sup>rd</sup>	19:30:23	19:35:23	5	none
July 4 <sup>th</sup>	19:30:17	19:35:17	5	After 2-1/2 years for nominal case
July 5 <sup>th</sup>	19:29:36	19:44:36	15	After 2-1/4 years for nominal case
July 16 <sup>th</sup>	20:23:59	20:28:59	5	After 3-1/2 years for +3-sigma case
July 17 <sup>th</sup>	20:18:59	20:38:59	20	After 3-1/2 years in all cases
July 18 <sup>th</sup>	20:23:57	20:38:57	15	After 3 years for +3-sigma
July 19 <sup>th</sup>	20:19:36	20:19:56	<1*	*The +3-sigma case fails after 2-1/2 years for the entire launch window. July 19 <sup>th</sup> becomes essentially an instantaneous

